Views of local function via decomposed sets

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Abstract. The paper deals with some new notions of local functions and their associated set operators. Induced topologies and their α sets, semi-open sets are discussed here.

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1. Introduction

Modern notation of Kuratowski [7] and Vaidyanathswami's local function is got new flavour when the mathematicians are discussed the same in new direction see [1, 3, 4, 5, 9, 10, 16, 18, 19]. In this present discussion we further consider the idea of local function and to determine new local function and its associated set function. Characterizations of the values of these two set operators are also discussed here. We also discuss the new topologies associated with these two operators according.

2. **Preliminaries**

Let † be a topology on a set Θ. That is, (Θ, †) (or simply Θ) is a topological space. Let $\vartheta \in \Theta$, then $\dagger(\vartheta)$ is denoted as the gathering of all open sets containing ϑ, whereas $\dagger^c(\vartheta)$ denotes the gathering of all closed sets containing ϑ. An ideal I on a set Θ is a non-empty collection of subsets of Θ, which satisfies the following properties: (i) if $A \in I$ and $B \subset A$, then $B \in I$ (ii) if $A, B \in I$, then $A \cup B \in I$. The triplecate (Θ, †, I) constitute an ideal topological space. For a subset A of Θ, the local function [7] of A in the ideal topological space (Θ, †, I), is denoted as A^* and defined as:
$A^*(I, \dagger) = \{\vartheta \in \Theta | U_\theta \cap A \not\subseteq I, \textit{ for each } U_\theta \in \dagger(\vartheta)\}$. We will denote $A^*(I, \dagger)$ by A^* or $A^*(I)$. Whereas the co-local function [20] of A with respect to I and † is as follows:
$A^\cdot(I, \dagger) = \{\vartheta \in \Theta | F_\theta \cap A \not\subseteq I, \textit{ for each } F_\theta \in \dagger^c(\vartheta)\}$.

3. On local functions

The section discuss the relationships among various local functions through the following examples.

Example 3.1. Let $\Theta = \{\vartheta_1, \vartheta_2, \vartheta_3\}$, $\dagger = \{\emptyset, \Theta, \{\vartheta_1\}, \{\vartheta_2\}, \{\vartheta_1, \vartheta_2\}\}$ and $I = \{\emptyset, \{\vartheta_1\}\}$. Then $\dagger^c = \{\emptyset, \Theta, \{\vartheta_3\}, \{\vartheta_2, \vartheta_3\}, \{\vartheta_1, \vartheta_3\}\}$. We consider $A = \{\vartheta_2, \vartheta_3\}$, Then $A^* = \{\vartheta_2, \vartheta_3\}$ and $A^\cdot = \{\vartheta_1, \vartheta_2, \vartheta_3\}$. Here, $\vartheta_1 \in A^\cdot$ but $\vartheta_1 \not\subseteq A^*$.

Example 3.2. In Example 3.1, if we take $A = \{\vartheta_1, \vartheta_3\}$, Then $A^* = \{\vartheta_3\}$ and $A^\cdot = \{\vartheta_1, \vartheta_2, \vartheta_3\}$. Here, $\vartheta_1 \in A^\cdot$ but $\vartheta_1 \not\subseteq A^*$. Also, $\vartheta_2 \in A^\cdot$ but $\vartheta_2 \not\subseteq A^*$.

Example 3.3. Let $\Theta = \{\vartheta_1, \vartheta_2, \vartheta_3\}$, $\dagger = \{\emptyset, \Theta, \{\vartheta_1\}, \{\vartheta_2\}, \{\vartheta_1, \vartheta_2\}\}$ and $I = \{\emptyset, \{\vartheta_1\}\}$. Then $\dagger^c = \{\emptyset, \Theta, \{\vartheta_3\}, \{\vartheta_2, \vartheta_3\}, \{\vartheta_1, \vartheta_3\}\}$. We consider $A = \{\vartheta_1, \vartheta_2\}$, Then $A^* = \{\vartheta_1, \vartheta_2, \vartheta_3\}$ and $A^\cdot = \{\vartheta_2\}$. Here, $\vartheta_1 \in A^*$ but $\vartheta_1 \not\subseteq A^\cdot$. Also, $\vartheta_3 \in A^*$ but $\vartheta_3 \not\subseteq A^\cdot$

Example 3.4. Let $\Theta = \{\vartheta_1, \vartheta_2, \vartheta_3\}$, $\dagger = \{\emptyset, \Theta, \{\vartheta_1\}, \{\vartheta_2\}, \{\vartheta_1, \vartheta_2\}\}$ and $I = \{\emptyset, \{\vartheta_1\}\}$. Then $\dagger^c = \{\emptyset, \Theta, \{\vartheta_3\}, \{\vartheta_2, \vartheta_3\}, \{\vartheta_1, \vartheta_3\}\}$. We consider $A = \{\vartheta_2\}$, Then $A^* = \{\vartheta_2, \vartheta_3\}$ and $A^\cdot = \{\vartheta_2\}$. Here, $\vartheta_1 \not\subseteq A^*$ and $\vartheta_1 \not\subseteq A^\cdot$.

Example 3.5. Let $\Theta = \{\vartheta_1, \vartheta_2, \vartheta_3\}$, $\dagger = \{\emptyset, \Theta, \{\vartheta_1\}, \{\vartheta_2\}, \{\vartheta_1, \vartheta_2\}\}$ and $I = \{\emptyset, \{\vartheta_1\}\}$. Then $\dagger^c = \{\emptyset, \Theta, \{\vartheta_3\}, \{\vartheta_2, \vartheta_3\}, \{\vartheta_1, \vartheta_3\}\}$. We consider $A = \{\vartheta_2, \vartheta_3\}$, Then $A^* = \{\vartheta_2, \vartheta_3\}$, $A^\cdot = \{\vartheta_1, \vartheta_2, \vartheta_3\}$, $A^{\cdot *} = \{\vartheta_2, \vartheta_3\}$ and $A^{*\cdot} = \{\vartheta_1, \vartheta_2, \vartheta_3\}$. Therefore, $A^{*\cdot} \not\subset A^{\cdot *}$.

Example 3.6. If we take $A = \{\vartheta_1, \vartheta_2\}$ in Example 3.5, then $A^* = \{\vartheta_1, \vartheta_2, \vartheta_3\}$, $A^\cdot = \{\vartheta_2\}$, $A^{\cdot *} = \{\vartheta_2, \vartheta_3\}$ and $A^{*\cdot} = \{\vartheta_1, \vartheta_2, \vartheta_3\}$. Therefore, $A^{*\cdot} \not\subset A^{\cdot *}$.

Example 3.7. If we take $A = \{\vartheta_1, \vartheta_2\}$ in Example 3.5, then $A^* = \{\vartheta_2, \vartheta_3\}$, $A^\cdot = \{\vartheta_2\}$, $A^{\cdot *} = \{\vartheta_2, \vartheta_3\}$ and $A^{*\cdot} = \{\vartheta_1, \vartheta_2, \vartheta_3\}$. Therefore, $A^{*\cdot} \not\subset A^{\cdot *}$.

4. Conflict of local function 1

Consider the decompose neighborhood of a point in a topological space Θ by, $\dagger_o^c(\vartheta) = \{U_\theta \cap F_\theta \mid U_\theta \in \dagger(\vartheta)\ \&\ F_\theta \in \dagger^c(\vartheta)\}$. Let $\mathbf{A} \subseteq \Theta$. We define $A^\dagger(\mathrm{I}, \dagger) = \{\vartheta \in \Theta \mid C_\theta \cap A \notin \mathrm{I},\ \forall\ C_\theta \in \dagger_o^c(\vartheta)\}$.

Theorem 4.1. *For the ideal* I *and the topology* † *on a set* Θ*,*

(i) $\emptyset^\dagger = \emptyset$.
(ii) $A^\dagger \subseteq B^\dagger$ *whenever* $A \subseteq B \subseteq \Theta$.
(iii) $(A \cup B)^\dagger = A^\dagger \cup B^\dagger$*, where* $A, B \subseteq \Theta$.
(iv) $(A^\dagger)^\dagger \subseteq A^\dagger$*, where* $A \subseteq \Theta$.
(v) $I^\dagger = \emptyset$*, whenever* $I \in \mathrm{I}$.
(vi) $(A \setminus I)^\dagger = A^\dagger = (A \cup I)^\dagger$*, where* $A \subseteq \Theta$ & $I \in \mathrm{I}$.
(vii) $A^\dagger \subseteq A^*$ & $A^\dagger \subseteq A^\cdot$*, where* $A \subseteq \Theta$.
(viii) $A^\dagger \setminus B^\dagger = (A \setminus B)^\dagger \setminus B^\dagger$*, where* $A, B \subseteq \Theta$.
(ix) $T \cap A^\dagger = T \cap (T \cap A)^\dagger \subseteq (T \cap A)^\dagger$*, where* T *is a closed set or an open set* & $A \subseteq \Theta$.
(x) $A^\dagger(\mathrm{I} \cap \mathrm{J}, \dagger) = A^\dagger(\mathrm{I}, \dagger) \cup A^\dagger(\mathrm{J}, \dagger)$*, where* J *is a more ideal on* Θ & $A \subseteq \Theta$.

Proof. (*iii*) Suppose $\vartheta \in (A \cup B)^\dagger$, but $\vartheta \notin A^\dagger$ & $\vartheta \notin B^\dagger$. Then there exist $U_\theta, U'_\theta \in \dagger_o^c(\vartheta)$ such that $U_\theta \cap A \in \mathrm{I}$ & $U'_\theta \cap B \in \mathrm{I} \implies U_\theta \cap U'_\theta \cap A \in \mathrm{I}$ & $U_\theta \cap U'_\theta \cap B \in \mathrm{I}$. Given that intersection of two members of $\dagger_o^c(\vartheta)$ is a again a member of $\dagger_o^c(\vartheta)$. Thus, $U_\theta \cap U'_\theta \cap (A \cup B) \in \mathrm{I}$, is a contradiction to the fact that $\vartheta \in (A \cup B)^\dagger$.

(*iv*) Suppose $\vartheta \in (A^\dagger)^\dagger$. Then $\forall\ U_\theta \in \dagger_o^c(\vartheta)$, $U_\theta \cap A^\dagger \notin \mathrm{I} \implies U_\theta \cap A^\dagger \neq \emptyset$, $\forall\ U_\theta \in \dagger_o^c(\vartheta)$. If $t \in U_\theta \cap A^\dagger$, then $U_\theta \cap A \notin \mathrm{I}$, as $U_\theta \in \dagger_o^c(t)$. Therefore, $\vartheta \in A^\dagger$.

(vii) Obvious from the fact that, $U_\theta \in \dagger(\vartheta) \implies U_\theta \in \dagger_o^c(\vartheta)$ & $F_\theta \in \dagger^c(\vartheta) \implies U_\theta \in \dagger_o^c(\vartheta)$.

(viii) It is obvious that $A^\dagger \setminus B^\dagger \subseteq (A \setminus B)^\dagger \setminus B^\dagger$. Now, let $t \in [(A \setminus B)^\dagger \setminus B^\dagger]$. This implies that $t \in (A \setminus B)^\dagger$ but $t \notin B^\dagger$. Implies that $t \in A^\dagger$ but $t \notin B^\dagger \implies t \in (A^\dagger \setminus B^\dagger)$. This property is free from the collection, that is, it is hold for any local function.

(ix) It is known that $(A \cap T)^\dagger \subseteq A^\dagger$. Its implies $T \cap (A \cap T)^\dagger \subseteq T \cap A^\dagger$. Now for closed set T, suppose $t \in T \cap A^\dagger$. Then, $t \in T$ & $t \in A^\dagger \implies \forall\ U_t \in \dagger_o^c(t)$, $U_t \cap A \notin \mathrm{I}$. As T is a closed set containing t, then for all $U_t \in \dagger_o^c(t)$, $U_t \cap T \in \dagger_o^c(t)$. Hence $U_t \cap T \cap A \notin \mathrm{I} \implies t \in (T \cap A)^\dagger$.

The proof will be similar when T is an open set.

Therefore, from above, the following operator induces a new topology.

Theorem 4.2. *Let* I *be an ideal on* Θ *and* † *be a topology on* Θ*. Then*

(i) $\mathrm{H}(A) = A \cup A^\dagger$ *for any* $A \subseteq \Theta$*, satisfies all the conditions of closure operator.*

(ii) *The open sets of the induced topology by the above operator* H *are:* $\{\mathbf{O} \subseteq \Theta \mid \mathrm{H}(\Theta \setminus \mathbf{O}) = \Theta \setminus \mathbf{O})\}$ *(=* $\dagger^{\uparrow}$ *say).*

(iii) *The gathering* $\partial_{\uparrow}(\mathrm{I}, \dagger) = \{O \setminus I \mid O \text{ is open or closed in } \Theta \ \&\ I \in \mathrm{I}\}$ *is a basis for* $\dagger^{\uparrow}$*.*

(iv) *For any* $A \subseteq \Theta$, $\mathrm{H}(A) \subseteq \mathrm{C}^*(A)$ *(where* $\mathrm{C}^*(A) = A \cup A^*$ *) and* $\mathrm{H}(A) \subseteq \mathrm{C}^{\cdot}(A)$ *(where* $\mathrm{C}^{\cdot}(A) = A \cup A^{\cdot}$*) hold.*

(v) $\dagger \subseteq \dagger^* \subseteq \dagger^{\uparrow}$ & $\dagger^{\cdot} \subseteq \dagger^{\uparrow}$ *hold.*

Proof. (*iii*) Claim: $\partial_{\uparrow}(\mathrm{I}, \dagger) \subseteq \dagger^{\uparrow}$

1. Suppose F is closed in Θ. Put $A = \Theta \setminus (F \setminus J)$, where $J \in \mathrm{I}$. Various expression of A is, $A = \Theta \setminus (F \setminus J) = \Theta \setminus [F \cap (\Theta \setminus J)] = (\Theta \setminus F) \cup J$.
2. Suppose $x \notin A$. Then $x \in F \setminus J$ and it follows that $x \in F$. Now $F \cap A = F \cap [(\Theta \setminus F) \cup J] = F \cap J \in \mathrm{I}$. This implies that $x \notin A^{\uparrow}$, and hence $A^{\uparrow} \subseteq A$.
3. A is closed in $\dagger^{\uparrow}$, and hence $(F \setminus J) \in \dagger^{\uparrow}$.
4. If U is open in Θ, then by the similar way, we have again $\partial_{\uparrow}(\mathrm{I}, \dagger) \subseteq \dagger^{\uparrow}$.

Finally, suppose $t \in V \in \dagger^{\uparrow}$. Then $t \notin (\Theta \setminus V) \supseteq (\Theta \setminus V)^{\uparrow}$. Then there exists a $U_t \in \dagger^c_\delta(t)$ such that $U_t \cap (\Theta \setminus V) \in \mathrm{I}$. Thus, $U_t \setminus V = J \in \mathrm{I}$, and hence $t \in U_t \setminus J \subseteq V$.

5. Conflict of local function 2

Define the un-decompose neighborhood of a point in Θ by, $\Delta\, \dagger^c_\delta(\vartheta) = \{U_\theta \cup F_\theta \mid U_\theta \in \dagger(\vartheta)\ \&\ F_\theta \in \dagger^c(\vartheta)\}$. Let $A \subseteq \Theta$. We define $A^{\downarrow}(\mathrm{I}, \dagger) = \{\vartheta \in \Theta \mid C_\theta \cap A \notin \mathrm{I},\ \forall\ C_\theta \in \Delta\, \dagger^c(\vartheta)\}$.

Theorem 5.1. *For the ideal* I *and the topology* $\dagger$ *on a set* Θ*,*

(i) $\emptyset^{\downarrow} = \emptyset$.

(ii) $A^{\downarrow} \subseteq B^{\downarrow}$ *whenever* $A \subseteq B \subseteq \Theta$.

(iii) $(A \cup B)^{\downarrow} = A^{\downarrow} \cup B^{\downarrow}$*, where* $A, B \subseteq \Theta$.

(iv) $(A^{\downarrow})^{\downarrow} \subseteq A^{\downarrow}$*, where* $A \subseteq \Theta$.

(v) $I^{\downarrow} = \emptyset$*, whenever* $I \in \mathrm{I}$.

(vi) $(A \setminus I)^{\downarrow} = A^{\downarrow} = (A \cup I)^{\downarrow}$*, where* $A \subseteq \Theta$ & $I \in \mathrm{I}$.

(vii) $A^* \subseteq A^{\downarrow}$, $A^{\cdot} \subseteq A^{\downarrow}$ & $A^{\uparrow} \subseteq A^{\downarrow}$*, where* $A \subseteq \Theta$.

(viii) $A^{\downarrow} \setminus B^{\downarrow} = (A \setminus B)^{\downarrow} \setminus B^{\downarrow}$*, where* $A, B \subseteq \Theta$.

(ix) $T \cap A^{\downarrow} = T \cap (T \cap A)^{\downarrow} \subseteq (T \cap A)^{\downarrow}$*, where* T *is a closed as well as an open set* & $A \subseteq \Theta$.

(x) $A^{\downarrow}(\mathrm{I} \cap \mathrm{J}, \dagger) = A^{\downarrow}(\mathrm{I}, \dagger) \cup A^{\downarrow}(\mathrm{J}, \dagger)$*, where* J *is a more ideal on* Θ & $A \subseteq \Theta$.

Proof. (*iii*) Suppose $\vartheta \in (A \cup B)^{\downarrow}$ but $\vartheta \notin A^{\downarrow}$ & $\vartheta \notin B^{\downarrow}$. Then there exist $U_\theta, U'_\theta \in \Delta\, \dagger^c_o(\vartheta)$ such that $U_\theta \cap A \in \mathrm{I}$ & $U'_\theta \cap B \in \mathrm{I}$. Then, $U_\theta \cap U'_\theta \cap A \in \mathrm{I}$ & $U_\theta \cap U'_\theta \cap B \in \mathrm{I} \implies U_\theta \cap U'_\theta \cap (A \cup B) \in \mathrm{I}$ (as $(W_\theta \cap W'_\theta) \cup (F_\theta \cap F'_\theta) \subseteq (W_\theta \cup F_\theta) \cap (W'_\theta \cup F'_\theta)$). We reached a contradiction to $\vartheta \in (A \cup B)^{\downarrow}$.

(*vii*) Last relation: Obvious from that fact that $U_\theta \cap F_\theta \cap A \notin \mathrm{I}$ implies $U_\theta \cup F_\theta \cap A \notin \mathrm{I}$

Theorem 5.2. *Local function in Theorem 5.1 satisfies the following:*

(i) *Define* $\int(A) = A \cup A^{\downarrow}$ *for any* $A \subseteq \Theta$*. Then* $\int$ *is a closure operator.*

(ii) *The open sets of the induced topology by the above operator* $\int$ *are:* $\{\mathbf{O} \subseteq \Theta \mid \int(\Theta \setminus \mathbf{O}) = \Theta \setminus \mathbf{O})\}$ *(=* $\dagger^{\downarrow}$ *say).*

(iii) *The gathering* $\partial_{\downarrow}(\mathrm{I}, \dagger) = \{O \setminus I \mid O$ *is open and closed in* Θ & $I \in \mathbf{I}\}$ *is a basis for* $\dagger^{\downarrow}$*.*

(iv) *For any* $A \subseteq \Theta$, $\mathrm{C}*(A) \subseteq \int(A)$, $\mathrm{C}^{\cdot}(A) \subseteq \int(A)$ & $\mathrm{H}(A) \subseteq \int(A)$ *hold.*

(v) $\dagger \subseteq \dagger^{\downarrow} \subseteq \dagger^{*} \subseteq \dagger^{\uparrow}$ & $\dagger^{\downarrow} \subseteq \dagger^{\cdot} \subseteq \dagger^{\uparrow}$ *hold.*

Proof. Proof is quite similar with the Theorem 4.2. In addition, we only add F is closed and open set in the last step of 4 in (*iii*).

6. Njåstad's α set of $\dagger^{\cdot}$, $\dagger^{\uparrow}$ & $\dagger^{\downarrow}$

In this section we shall consider the following operators to catch the α set of the topologies $\dagger^{\cdot}$, $\dagger^{\uparrow}$ & $\dagger^{\downarrow}$.

For the topology $\dagger$ and the ideal I on a set Θ, we mentioned that $\perp_{\cdot}(A) = \Theta \setminus (\Theta \setminus A)^{\cdot}$, $\perp_{\uparrow}(A) = \Theta \setminus (\Theta \setminus A)^{\uparrow}$ & $\perp_{\downarrow}(A) = \Theta \setminus (\Theta \setminus A)^{\downarrow}$.

The gathering, $\{A \subseteq \Theta \mid A \subseteq ((\perp_{\cdot}(A))^{-})^{0}\}$ (= $\dagger^{\perp\cdot}$ say), $\{A \subseteq \Theta \mid A \subseteq (\perp_{\uparrow}(A))^{-})^{0}\}$ (= $\dagger^{\perp\uparrow}$ say) & $\{A \subseteq \Theta \mid A \subseteq ((\perp_{\downarrow}(A))^{-})^{0}\}$ (= $\dagger^{\perp\downarrow}$ say) measure the Njåstad's α set of the respective topologies, where $-$ and 0 are mentioned as the closure and interior operators respectively.

Theorem 6.1. *For an ideal* I & *and a topology* $\dagger$ *on the set* Θ*, following hold:*

(i) $\dagger^{\perp\cdot}$ *form a topology on* Θ*, when* $\dagger^{c} \cap \mathrm{I} = \{\emptyset\}$*.*

(ii) $\dagger^{\perp\uparrow}$ *form a topology on* Θ*, when* $\dagger \cap \mathrm{I} = \{\emptyset\}$*.*

(iii) $\dagger^{\perp\downarrow}$ *form a topology on* Θ*, when* $\dagger \cap \mathrm{I} = \{\emptyset\}$*.*

Lemma 6.1. *The set* Θ *with ideal* I & *topology* $\dagger$ *satisfying following:*

(i) $\perp_{\cdot}(A) \neq \emptyset \Longleftrightarrow A$ *contains a* $\dagger^{\cdot}$*-interior, when* $\dagger^{c} \cap \mathrm{I} = \{\emptyset\}$*.*

(ii) $\perp_{\uparrow}(A) \neq \emptyset \Longleftrightarrow A$ *contains a* $\dagger^{\uparrow}$*-interior, when* $\dagger \cap \mathrm{I} = \{\emptyset\}$*.*

(iii) $\perp_{\downarrow}(A) \neq \emptyset \Longleftrightarrow A$ *contains a* $\dagger^{\downarrow}$*-interior, when* $\dagger \cap \mathrm{I} = \{\emptyset\}$*.*

Lemma 6.2. *The set* Θ *with ideal* I & *topology* $\dagger$ *satisfying following:*

(i) $\{\vartheta\} \in SO(\dagger^{\cdot}) \Longleftrightarrow \{\vartheta\} \in \dagger^{\perp\cdot}$.

(ii) $\{\vartheta\} \in SO(\dagger^{\uparrow}) \Longleftrightarrow \{\vartheta\} \in \dagger^{\perp\uparrow}$.

(iii) $\{\vartheta\} \in SO(\dagger^{\downarrow}) \Longleftrightarrow \{\vartheta\} \in \dagger^{\perp\downarrow}$.

Theorem 6.2. *The set* Θ *with ideal* I & *topology* $\dagger$ *satisfying following:*

(i) $\dagger^{\perp\cdot}$ *is exactly collection such that* $A \in \dagger^{\perp\cdot}$ & $B \in SO(\dagger^{\cdot}) \Longrightarrow A \cap B \in SO(\dagger^{\cdot})$*, when* $\dagger^{c} \cap \mathrm{I} = \{\emptyset\}$*.*

(ii) $\dagger^{\perp\uparrow}$ *is exactly collection such that* $A \in \dagger^{\perp\uparrow}$ & $B \in SO(\dagger^{\uparrow}) \Longrightarrow A \cap B \in SO(\dagger^{\uparrow})$*, when* $\dagger \cap \mathrm{I} = \{\emptyset\}$*.*

(iii) $\dagger^{\perp\downarrow}$ *is exactly collection such that* $A \in \dagger^{\perp\downarrow}$ & $B \in SO(\dagger^{\downarrow}) \Longrightarrow A \cap B \in SO(\dagger^{\downarrow})$*, when* $\dagger \cap \mathrm{I} = \{\emptyset\}$*.*

Theorem 6.3. *For the topology* $\dagger$ *on the set* Θ*, α sets,* $\dagger^a$ *consists of exactly those sets* A *for which* $A \cap B \in SO(\dagger)$ *for all* $B \in SO(\dagger)$*. So* $\dagger^a$ *is a topology on* Θ*, known as α-topology.*

Theorem 6.4. *For the topology* $\dagger$ & *the ideal* I *on the set* Θ*, following holds:*

(i) $\dagger^{\perp\cdot} = (\dagger^{\cdot})^a$*, when* $\dagger^c \cap \mathrm{I} = \{\emptyset\}$.

(ii) $\dagger^{\perp\uparrow} = (\dagger^{\uparrow})^a$*, when* $\dagger \cap \mathrm{I} = \{\emptyset\}$.

(iii) $\dagger^{\perp\downarrow} = (\dagger^{\downarrow})^a$*, when* $\dagger \cap \mathrm{I} = \{\emptyset\}$.

Further expression α sets in the various topologies are given bellow:

For the ideal topological space $(\Theta, \dagger, \mathrm{I})$, $\mathrm{I}^{\sim} = \{A \subseteq \Theta \mid (A*)^0 = \emptyset\}$ is further an ideal. Then ideal makes following equivalent conditions:

Lemma 6.3. *For the ideal topological space* $(\Theta, \dagger, \mathrm{I})$*,*

(i) $\dagger \cap \mathrm{I} = \{\emptyset\}$*;*

(ii) $\dagger \cap \mathrm{I}^{\sim} = \{\emptyset\}$*;*

(iii) $\mathrm{I}^{\sim}$ *is the same as the gathering of nowhere dense sets of* $\dagger^{\cdot}$ *(resp.* $\dagger^{\uparrow}$ & $\dagger^{\uparrow}$*);*

are identical.

Theorem 6.5. *For an ideal* I *and a topology* $\dagger$ *on the set* Θ*, we have:*

(i) $\dagger^{\perp\cdot} = \{O \setminus N \mid O \in \dagger^{\cdot},\ N \in \mathrm{I}^{\sim}\}$.

(ii) $\dagger^{\perp\uparrow} = \{O \setminus N \mid O \in \dagger^{\uparrow},\ N \in \mathrm{I}^{\sim}\}$.

(iii) $\dagger^{\perp\downarrow} = \{O \setminus N \mid O \in \dagger^{\downarrow},\ N \in \mathrm{I}^{\sim}\}$.

The proof of the results of section follows from [2, 6, 8, 11, 12, 13, 14, 15, 17].

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